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Research article

Numerical Simulation of Granular Mixing in Conical Mixer with Different Angles Using Discrete Element Method

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Abstract: Granular mixing process is necessary for operation in many industries such as food, pharmaceutical and cosmetic industries. Although the mixing materials are almost the first stage in the production process, it can affect the quality and cost of the final product. They are choosing a suitable mixer that may reduce cost and energy consumption. A static mixer is a device widely used to mix materials in industries. Because the mixing phenomena are very complex, a proper static mixer design still needs to be studied. In this work, we simulate the mixing process of granular materials in a conical mixer. We assume the two types of granular solid with the same size flowing in the mixer. The discrete Element Method (DEM) is applied to obtain numerical solutions in the mathematical model. This work aims to use a coordinate mixing index to compare the effectiveness of mixing from the conical mixers with different angles, including 15°, 30°, 45° and 60°. The results show that the highest coordinate mixing number occurs in the mixer with an angle 15°. These imply that it is the best mixing quality.

Keywords: Discrete Element Method (DEM); Coordinate Mixing Index; Conical Mixer; Granular Material Mixing.

1. Introduction

The mixing process is widely applied in light and heavy industries such as chemical, pharmaceutical, food, and fertilizer. Researchers have continuously presented the development of the mixing method. Over a few decades, a static mixer has been widely used in many industries because it is a motionless mixer and consumes low energy. However, new geometric designs of static mixers have been developed to increase mixing efficiency. Many researchers have studied the granular mixing process with a static mixer. In the earlier study, research focuses on an experiment to investigate the granular mixing phenomena [1-4]. However, the studies cannot be completed to describe behaviours of granular mixing because there are different blenders and structures in various industries. According to the problems, numerical simulations have been applied to overcome the limitations of experimental study. A model based on the Discrete Element Method (DEM) has been used to capture the collision of particles.

Many researchers have used numerical simulation based on DEM to study the granular behaviour during movement. It was used to capture particle movement and energy distribution of collision in two types of impact crusher [6-8] and many research groups had done the same work involving grinding [5-6]. Some groups have used DEM simulation to simulate granular flow in hopper [8-12]. Moreover, it has been used in granular mixing [7-8, 13-16]. It is agreed that DEM is a good tool in capturing the particle collision and predicting the flow phenomena [17-20]. For granular mixing, the simulation's objectives are to study the factors that affect mixing speed and mixing quality.

In this paper, we aim to optimize the geometry of static mixer devices. The DEM is used to simulate the mixing process of two type's granular materials of the same size in four different angles of the conical mixer. A coordinated mixing index is used to obtain mixing efficiency.

2. Mathematical Model

The discrete Element Method was presented in 1979 by Cundall and Strack [20]. It was used for the investigation of granular material. In the method, we need to identify the forces acting on the particle i due to contact of particle-particle and particle-wall. The contact between particles i and j is defined by

$$\delta = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \leq R_i + R_j, \tag{1}$$

where R_i and R_j are the radii of the particles i and j. $\left(x_i, y_i\right)$ and $\left(x_j, y_j\right)$ are the center point of the particles i and j, respectively, as shown in Figure 1.

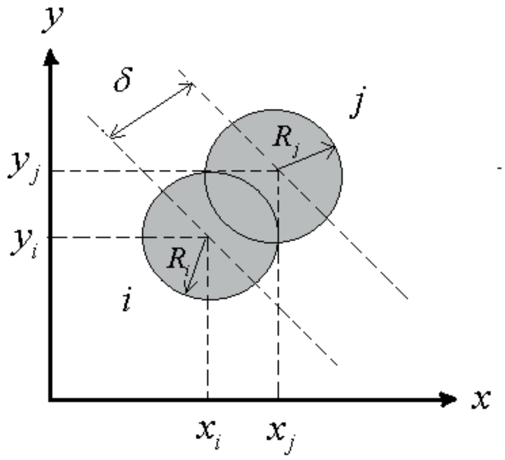


Figure 1. Two particles i and j are in contact.

To obtain a numerical solution, we consider when the particle i contacts other particles or walls. Forces and moments are acting on it. The numerical integration of Newton's equation of motion is employed to calculate new velocities and positions of all particles. By Newton's law, the governing equations of translational and rotational movement of a particle i are shown below

$$m_i \frac{d^2\mathbf{r}_i(\mathrm{t})}{dt^2} = \mathbf{F}_{gi} + \sum_{j=1, j\neq i}^{n_p} \mathbf{F}_{c,ij} + \sum_{k=1}^{n_w} \mathbf{F}_{c,iw_k}, \tag{2}$$

$$I_i \frac{d^2\boldsymbol{\theta}_i(\mathrm{t})}{dt^2} = \sum_{j=1, j\neq i}^{n_p} \mathbf{r}_i \times \mathbf{F}_{c,ij} + \sum_{k=1}^{n_w} \mathbf{r}_i \times \mathbf{F}_{c,iw_k}, \tag{3}$$

where m_i and I_i are mass and rotation moment of inertia of particle i, respectively. $\mathbf{r}_i(\mathrm{t})$ and $\boldsymbol{\theta}_i(\mathrm{t})$ represent vectors of position and angular rotation at the centre of the particle i, respectively. $\frac{d^2\mathbf{r}_i(\mathrm{t})}{dt^2}$ and $\frac{d^2\boldsymbol{\theta}_i(\mathrm{t})}{dt^2}$ are translational acceleration and rotational acceleration vectors of the center of the particle i, respectively. $\mathbf{F}_{gi}$ is gravitational forces, $\mathbf{F}_{c,ij}$ and $\mathbf{F}_{c,iw_k}$ are contact forces due to particle j and wall w_k, respectively.

The total forces acting on the particle i can be written as follows

$$\mathbf{F}_{gi} + \sum_{j=1,\, j\neq i}^{n_p} \mathbf{F}_{c,ij} + \sum_{k=1}^{n_w} \mathbf{F}_{c,iw_k} = \mathbf{F}_{c,i}, \tag{4}$$

$$\sum_{j=1,\, j\neq i}^{n_p} \mathbf{r}_i \times \mathbf{F}_{c,ij} + \sum_{k=1}^{n_w} \mathbf{r}_i \times \mathbf{F}_{c,iw_k} = \mathbf{M}_{c,i}. \tag{4}$$

Therefore, equations (2) and (3) become

$$m_i \frac{d^2\mathbf{r}_i(\mathrm{t})}{dt^2} = \mathbf{F}_{c,i}, \tag{6}$$

$$I_i \frac{d^2\boldsymbol{\theta}_i(\mathrm{t})}{dt^2} = \mathbf{M}_{c,i}. \tag{7}$$

Let

$$\mathbf{v}_i(\mathrm{t}) = \frac{d\mathbf{r}_i(\mathrm{t})}{dt}, \tag{8}$$

$$\boldsymbol{\omega}_i(\mathrm{t}) = \frac{d\boldsymbol{\theta}_i(\mathrm{t})}{dt}. \tag{9}$$

Let n_p and n_w are the number of particles and walls in the simulation.

For $i = 1, 2, 3, ..., n_p$, we get a system of first-order ordinary differential equation, as shown below

$$\frac{d\mathbf{r}_i(\mathrm{t})}{dt} = \mathbf{v}_i(\mathrm{t}),$$

$$\frac{d\boldsymbol{\theta}_i(\mathrm{t})}{dt} = \boldsymbol{\omega}_i(\mathrm{t}),$$

$$\frac{d\mathbf{v}_i(\mathrm{t})}{dt} = \frac{\mathbf{F}_{c,i}(\mathrm{t})}{m_i}, \tag{10}$$

$$\frac{d\boldsymbol{\omega}_i(\mathrm{t})}{dt} = \frac{\mathbf{M}_{c,i}(\mathrm{t})}{I_i}.$$

Let

$$\boldsymbol{\psi}(\mathrm{t}) = \begin{bmatrix} \mathbf{r}_1(\mathrm{t}) & \boldsymbol{\theta}_1(\mathrm{t}) & \mathbf{r}_2(\mathrm{t}) & \boldsymbol{\theta}_2(\mathrm{t}) & \cdots & \mathbf{r}_{n_p}(\mathrm{t}) & \boldsymbol{\theta}_{n_p}(\mathrm{t}) \end{bmatrix}^T,$$

$$\boldsymbol{\Phi}(\mathrm{t}) = \begin{bmatrix} \mathbf{v}_1(\mathrm{t}) & \boldsymbol{\omega}_1(\mathrm{t}) & \mathbf{v}_2(\mathrm{t}) & \boldsymbol{\omega}_2(\mathrm{t}) & \cdots & \mathbf{v}_{n_p}(\mathrm{t}) & \boldsymbol{\omega}_{n_p}(\mathrm{t}) \end{bmatrix}^T,$$

and

$$\mathbf{F}(\mathrm{t}) = \begin{bmatrix} \frac{\mathbf{F}_{c,1}(\mathrm{t})}{m_1} & \frac{\mathbf{M}_{c,1}(\mathrm{t})}{I_1} & \frac{\mathbf{F}_{c,2}(\mathrm{t})}{m_2} & \frac{\mathbf{M}_{c,2}(\mathrm{t})}{I_2} & \cdots & \frac{\mathbf{F}_{c,n_p}(\mathrm{t})}{m_{n_p}} & \frac{\mathbf{M}_{c,n_p}(\mathrm{t})}{I_{n_p}} \end{bmatrix}^T$$

for $i = 1, 2, 3, ..., n_p$.

Therefore, we obtain a system of ordinary differential equations as follows

$$\frac{d\boldsymbol{\psi}(\mathrm{t})}{dt} = \boldsymbol{\Phi}(\mathrm{t}), \tag{11}$$

$$\frac{d\boldsymbol{\Phi}(\mathrm{t})}{dt} = \mathbf{F}(\mathrm{t}). \tag{12}$$

The central difference scheme is applied. $\mathbf{F}_{c,i}, \mathbf{M}_{c,i}, \mathbf{v}_i, \boldsymbol{\omega}_i, \frac{d\mathbf{v}_i(\mathrm{t})}{dt}$ and $\frac{d\boldsymbol{\omega}_i(\mathrm{t})}{dt}$ are considered as constants in time interval $\Delta t = \left[t_{n-\frac{1}{2}}, t_{n+\frac{1}{2}} \right]$, equation (12) becomes

$$\frac{\boldsymbol{\Phi}^{n+\frac{1}{2}} - \boldsymbol{\Phi}^{n-\frac{1}{2}}}{\Delta t} = \mathbf{F}^n, \tag{13}$$

thus

$$\boldsymbol{\Phi}^{n+\frac{1}{2}} = \boldsymbol{\Phi}^{n-\frac{1}{2}} + \mathbf{F}^n (\Delta t). \tag{14}$$

The central difference scheme is applied to equation (11) at time $t_{n+\frac{1}{2}}$ we get

$$\frac{\boldsymbol{\psi}^{n+1} - \boldsymbol{\psi}^n}{\Delta t} = \boldsymbol{\Phi}^{n+\frac{1}{2}}, \tag{15}$$

thus

$$\boldsymbol{\psi}^{n+1} = \boldsymbol{\psi}^n + \boldsymbol{\Phi}^{n+\frac{1}{2}} (\Delta t), \tag{16}$$

$$\boldsymbol{\psi}^{n+1} = \boldsymbol{\psi}^{n} + \boldsymbol{\Phi}^{n-\frac{1}{2}}\left(\Delta t\right) + \mathbf{F}^{n}\left(\Delta t\right)^{2}. \tag{17}$$

From equations (14) and (17), we get new translational and rotational displacements at time $t = n+1$. It can be calculated by

$$\frac{d\boldsymbol{\psi}^{n+1}}{dt} = \boldsymbol{\Phi}^{n+1}. \tag{18}$$

3. Results and Discussion

This section presents the results, including velocity field, flow pattern, and mixing quality of granular particles from the simulation. Four conical mixers with 15°, 30°, 45° and 60° angle are chosen to compare mixing effectiveness, as shown in Figure 2. We investigate mixing two types of particles that are the same size and each of the types are different colours. All of the particles are supposed to be spherical shape. The parameter values used in the model are presented in Table 1. In the simulation, the number of particles is 3,000 particles. Positions of a particle at $t = 0s$ are shown in Figure 2. The numerical solution of particle flow in a conical mixer with four different angles is determined using the DEM method. The simulation begins when particles are discharged into a conical mixer.

Table 1. Model parameters.

Parameter	Value
Conical mixer height, $\mathrm{H}(cm)$	80
Conical mixer width, $\mathrm{W}(cm)$	40
Outlet width of conical mixer, $d(cm)$	4
Angle of conical mixer, θ	$15^{\circ}, 30^{\circ}, 45^{\circ}$ and 60°
Particle diameter,	0.75
Particle density, ρ (g/cm^{3})	1.003
Number of particles	3,000
Time step, $\Delta t, (\mathrm{s})$	5×10^{-6}

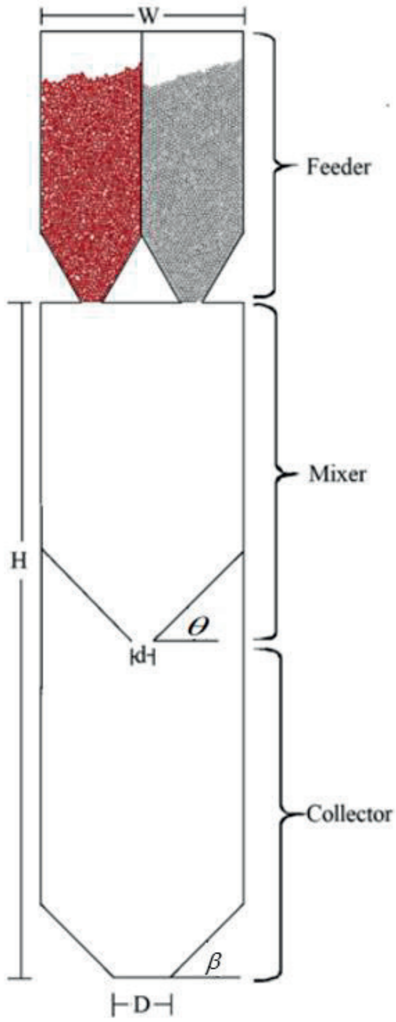


Figure 2. The geometry of the conical mixer with an angle of θ degree.

Figure 3 represents the distribution of particles when they fall in the mixers at $t = 60\ s$. The slowest mixing process can be found in a conical mixer with an angle of 15 degree. The velocity fields evaluated from the simulations are shown in Figure 4. It reveals that the velocity fields are well captured inside the mixer. For four mixers, the highest velocity of the particle occurs at the top of the collector zone. The conical mixer gives the lowest velocity. Figure 5 shows the final process of particle mixing in a conical mixer with an angle 45°.

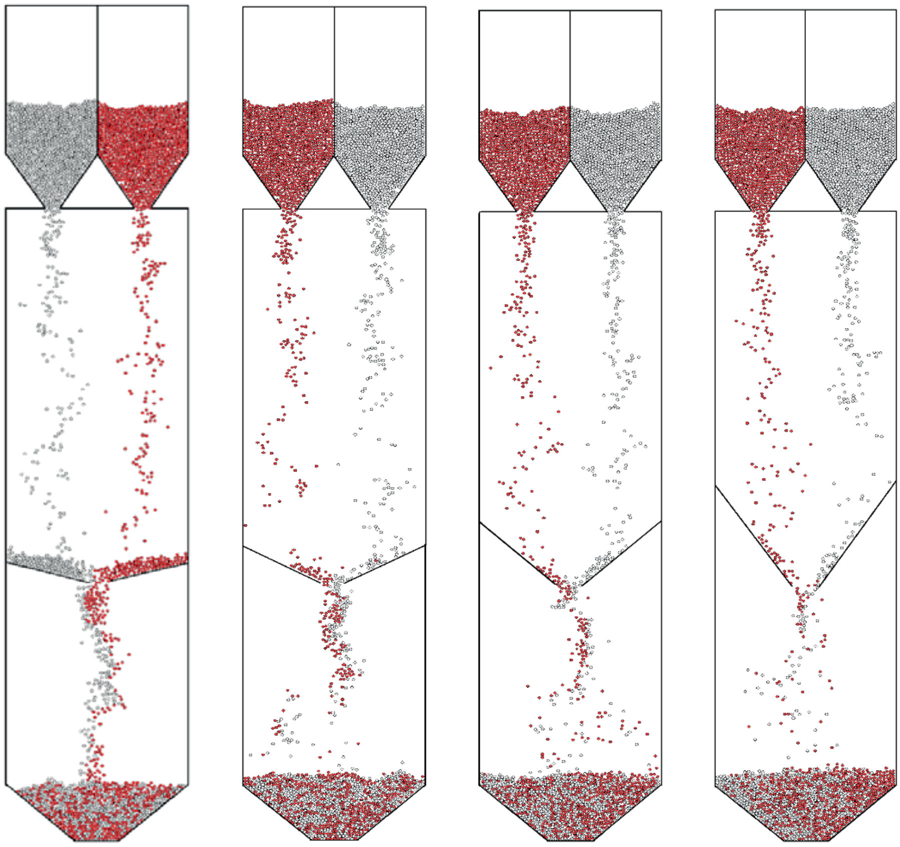


Figure 3. Distribution of all particles for four different angle geometries at $t = 60\ s$: (a) $\theta = 15^{\circ}$; (b) $\theta = 30^{\circ}$; (c) $\theta = 45^{\circ}$; (d) $\theta = 60^{\circ}$.

Table 2. Value of coordinate mixing index at the bottom part of the collector zone.

Angle (θ)	**Coordinate mixing index (M)**
15°	0.8120
30°	0.6750
45°	0.6332
60°	0.6786

In previous studies, the coordinate mixing index has been applied to evaluate the mixing quality [21-22]. This simulation uses the coordinate mixing index to calculate mixing quality. The coordinate mixing index is defined as

$$M = \frac{\Omega}{\Omega_{ran}}, \tag{19}$$

where Ω is the coordinate number of the system and Ω_{ran} denotes the coordinate number of the well-mixed system. The values of the coordinate mixing index are presented in Table 2. The comparison reveals that the conical mixer with 15° the highest coordinate mixing index, implying the best mixing quality.

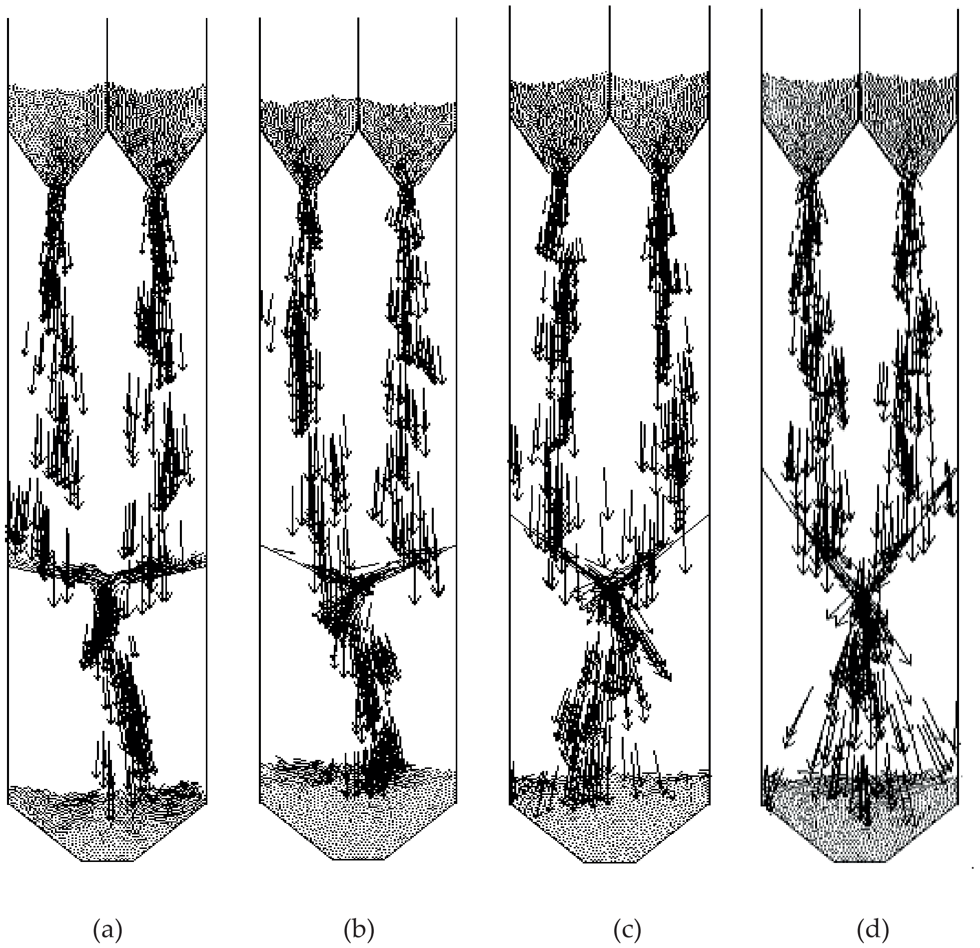
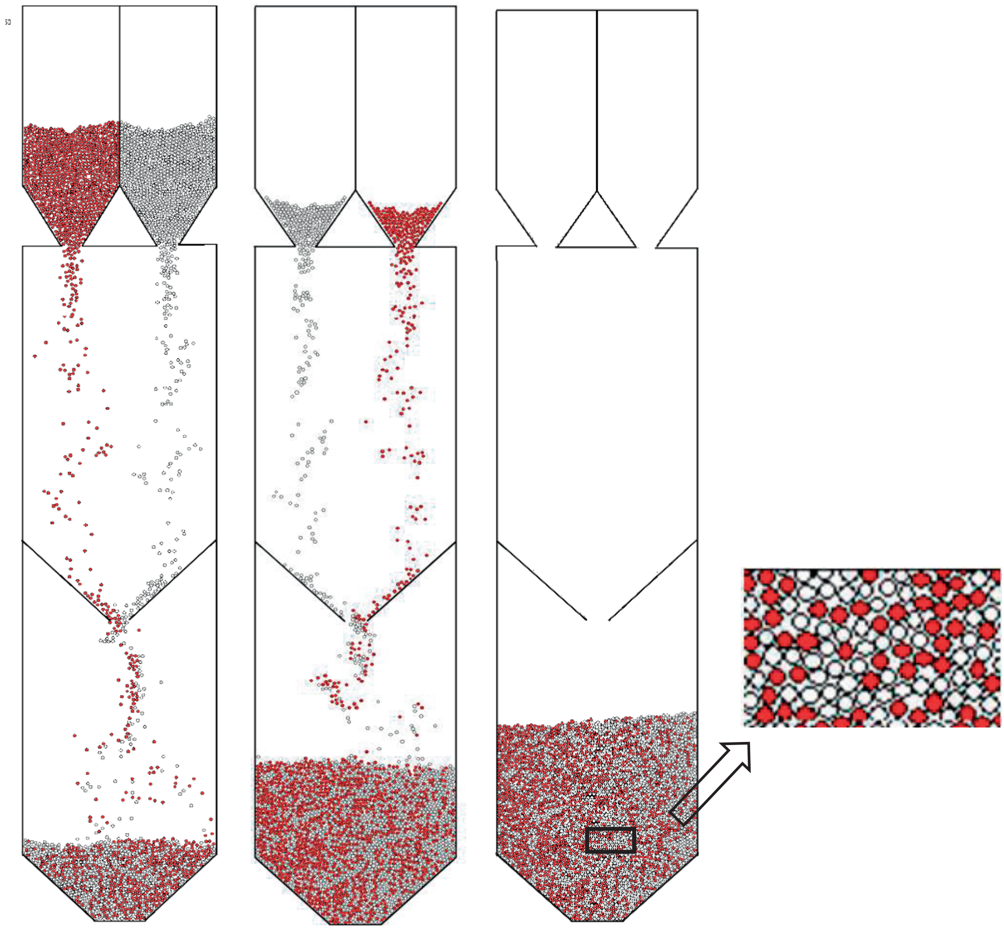


Figure 4. Velocity fields of granular material obtained from four different angle geometries at $t = 60\ s$ a) $\theta = 15^\circ$; (b) $\theta = 30^\circ$; (c) $\theta = 45^\circ$; (d) $\theta = 60^\circ$.



(a) (b) (c)

Figure 5. The final process of particle mixing is obtained from a conical mixer at different times: (a) $t = 50s$, (b) $t = 100s$ and (c) $t = 125s$.

4. Conclusions

This paper aims to optimize the mixing quality of granular materials in a conical mixer. The mathematical model based on a DEM is applied to simulate the mixing process. Newton's law determines the interaction of particles inside the mixer. In the simulation, the four different angles of the mixers with an angle of $15^\circ, 30^\circ, 45^\circ$ and 60° are studied the mixing quality. The velocity fields, positions of particles, and coordinate mixing index are calculated and compared. The results indicate that the mixing quality is higher for a conical mixer with an angle 15° than others. It is concluded that a conical mixer with the angle 15° contributes the best mixing quality.

Author Contributions: The first author formulated a mathematical model, simulated the results. The second author contributed to editing and revising the manuscript. The last author proposed the research idea for finding the numerical simulation and was responsible for examining simulation results. All authors read and approved the final manuscript.

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Conflicts of Interest: The authors declare that they have no competing interests.

References

1. Rafiee, M.; Simmons, M.J.H; Ingram, A.; Hugh, S.H. Development of position emission particle tracking for studying laminar mixing in Kenics mixer. *Chemical Engineering Research and Desing*. 2013, 91, 2106-2113.
2. Ghanem, A.; Lemenand, T.; Della, V.D.; Peerhossaini, H. Static mixers: mechanisms, applications, and characterization methods - a review. *Chemical Engineering Research and Desing*. 2014, 92(2), 205-228.
3. Baumann, A.; Jeelani, S.A.K.; Holenstein, B.; Windhab, E.J. Flow regimes and drop break-up in SMX and packed bed static mixers. *Chemical Engineering Science*, 2012, 73, 354-365.
4. Yang, H.C. Mixing Characteristics of Motionless Mixers. *Journal of Visuallization*, 2007, 10(1), 83-89.
5. Moakher, M.; Shinbrot, T.; Muzzio, F.J. Experimentally validated computations of flow, mixing and segregation of non-cohesive grains in 3D tumbling blenders. *Powder Technology*, 2000, 109, 58-71.
6. Michele, M.; Edmund, H.S. Discrete Element Method (DEM) for Industrial Applications: Comments on Calibration and Validation for the Modelling of Cylindrical Pellets. *KONA Powder and Particle Journal*. 2015, 32, 236-252.
7. Alian, M.; Ein-Mozaffari, F.; Upreti, S.R. Analysis of the mixing of solid particles in a plowshare mixer via discrete element method (DEM). *Powder Technology*, 2015, 274, 77-87.
8. Liu, X.; Hu, Z.; Wu, W.; Zhan, J.; Herz, F.; Specht, E. DEM study on the surface mixing and whole mixing of granular materials in rotary drums. *Powder Technology*, 2017, 315, 438-444.
9. Chuayjan, W.; Thongnak, S.; Wiwatanapataphee, B. The effect of inverted cone inserts design on granular behavior in silo. *Chamchuri Jouranl of Mathematics*. 2016, 8, 18-33.
10. Chuayjan, W.; Jumpen, W.; Boonkrong, P.; Wiwatanapataphee, B. Two-Phase flow of air and soybean during a silo discharging process. *International Journal of Mathematics Models and Method in Applied Sciences*, 2012, *8*(6), 909-916.
11. Chuayjan, W.; Boonkrong, P.; Wiwatanapataphee, B. Effect of the silo-bottom design on the granular behavior during discharging process. *Advamces in Intelligent Systems and Computing*, 2012, *62*(3), 68-73.
12. Chuayjan, W.; Pothiphan, S.; Wiwatanapataphee, B.; Wu, Y.H. Numerical simulation of granular flow during filling and discharging of a silo. *International Journal Pure and Applied Mathematics,* 2010, 62(3), 347-364.
13. Alian, M.; Ein-Mozaffari, F.; Upreti, S.R.; Wu, J. Using discrete element method to analyze the mixing of the solid particles in a slant cone mixer. *Chemical Engineering Research and Design*, 2015, 93, 318-329.
14. Bauman, I. Solid-Solid Mixing with Static Mixers. *Chemical and Biochemical Engineering Quarterly*, 2001, *15*(4), 159-165.
15. Yang, H.C. Mixing Characteristics of Motionless Mixers. *Journal of Visuallization*, 2007, *10*(1), 83-89.
16. Lemieux, M.; Leonard, G.; Doucet, J.; Leclaire, L. -A.; Viens, F.; Chaouki, J.; Bertrand, F. Large-scale numerical investigation of solids mixing in V-blender usiing the discrete element method. *Powder Technology*, 2008, 181, 205-216.
17. Zhu, H.P.; Zhou, ZY; Yang, R.Y.; Yu, A.B. Discrete particle simulation of particulate systems: theoretical developments. *Chemical Engineering Science*, 2007, 63(13), 3378-3396.
18. Wenqi, Z.; Aibing, Y.; Xuejiao, I.; Zhenbo, T.; Hao, Z. DEM/CFD-DEM modeling of non-spherical particulate systems: theoretical developments and applications. *Powder Technol*, 2016, 302, 108-152.
19. Bertrand, F.; Leclaire, L. -A.; Levecque, G. DEM-based models for the mixing of granular materials. *Chemical Engineering Science*, 2005, 60, 2517-2531.
20. He, S.; Gan, J.; Pinson, D.; Yu, A.; Zhou, Z. A Discrete Element Method Study of Monodisperse Mixing of Ellipsoidal Particles in a Rotating Drum. *Industrial & Engineering Chemistry Research*, 2020, 59, 12458 – 12470.
20. Cundall, P.A.; Strack, O.D.L. A discrete numerical model for granular assemblies. *Geotechnique*, 1979, 29, 47-65.
21. Lu, L.S.; Hsiau, S.S. Mixing in vibrated granular beds with the effect of electrostatic force. *Powder Technology*, 2005, 170-179.
22. Chou, S.H.; Song, Y.L.; Hsiau, S.S. A Study of the Mixing Index in Solid Particles. *KONA Powder and Particle Journal*, 2017, 34, 275-281.